Re-exam, 28th April 2014, PPP

3) 1. Δ^{++}: uuu all $\uparrow\uparrow\uparrow$ configuration

→ Pauli exclusion principle violated?

→ Solution: color label (RGB)

$$\psi_\Delta = \psi_{flavor} \cdot \psi_{spin} \cdot \psi_{space} \cdot \psi_{color}$$

even, even, even $(-1)^L$, hence odd!

→ antisym.

RGB − GRB + GBR − BGR etc.

2. Use Lorentz Invariance:

$$E_{lab,tot}^2 - P_{lab,tot}^2 = E_{cm,tot}^2 - \underbrace{P_{cm,tot}^2}_{=0} = M_\Delta^2$$

$$(E_\pi + M_p)^2 - P_\pi^2 = M_\Delta^2$$

$$E_\pi^2 + M_p^2 + 2M_pE_\pi - P_\pi^2 = M_\Delta^2$$

$$M_\pi^2 + M_p^2 + 2M_pE_\pi = M_\Delta^2 \Rightarrow E_\pi = \frac{M_\Delta^2 - M_\pi^2 - M_p^2}{2M_p}$$

$$= 329,8 \text{ MeV}$$

$$\Rightarrow P_\pi = \sqrt{E_\pi^2 - M_\pi^2} = 299 \text{ MeV/c}$$

(3) cont^{d}

3. $\Delta \longrightarrow \pi + N$ (the angular momentum L is between π and N)

$I(J^P): \frac{3}{2}(\frac{3}{2}^+) \longrightarrow 1(0^-) + \frac{1}{2}(\frac{1}{2}^+)$

$P_{\text{ARITY}} = P_\pi \cdot P_N \cdot (-1)^L = (-1)^{L+1} \Rightarrow L$ odd

$J = L + S$:

$L = 1 : 1 + \frac{1}{2} = (\frac{1}{2}, \underline{\underline{\frac{3}{2}}})$ O.K.

$L = 3 : 3 + \frac{1}{2} = (\frac{5}{2}, \frac{7}{2}) \Rightarrow$ No

etc

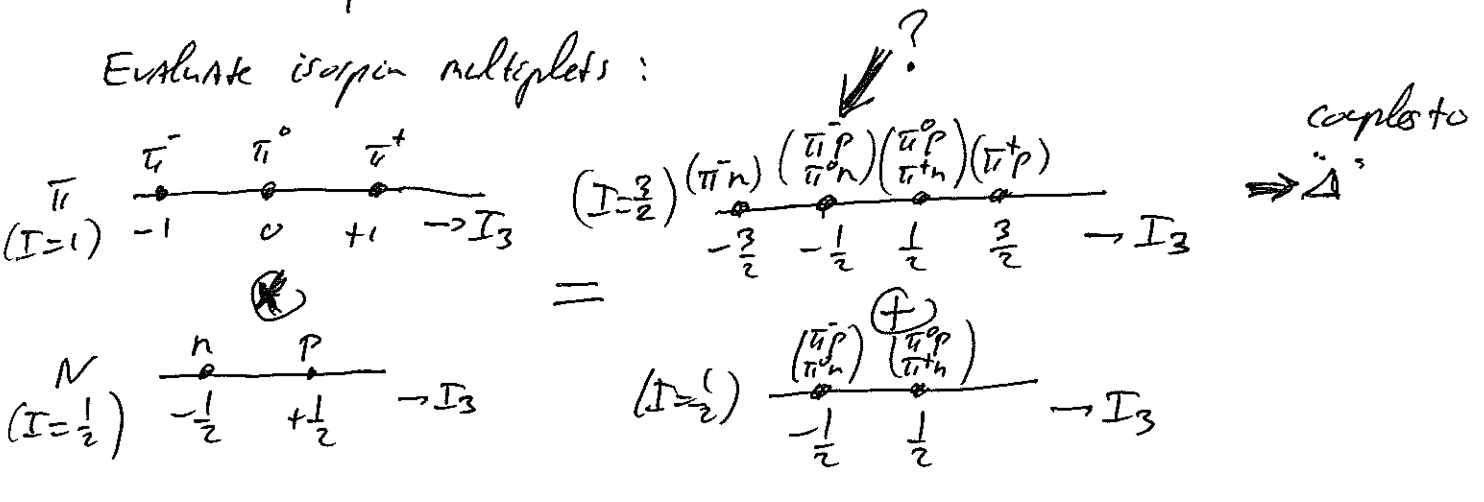
$L=1$ possible

P-wave.

4. $\Delta^0 \to \pi^0 + n$

$\to \pi^- + p$

Evaluate isospin multiplets :



$T_+ |\pi^- n\rangle = (T_{+,\pi} + T_{+,N}) |\pi^- n\rangle$

$= [\underline{\underline{\sqrt{2}}} |\pi^0 n\rangle + \underline{\underline{1}} \cdot |\pi^- p\rangle] / \sqrt{3}$ ⟸ normalization

$\left.\begin{array}{l} \langle \Delta^0 | \pi^0 n \rangle^2 = \frac{2}{3} \\ \langle \Delta^0 | \pi^- p \rangle^2 = \frac{1}{3} \end{array}\right\}$ Branching fractions B

3) cont'd

$$\sigma = \left(\frac{2\pi}{p^2}\right) \frac{\Gamma_{\Delta\to\pi^- p}\,\Gamma_{\Delta\to\pi^0 n}}{(E_{cm}-M_\Delta)^2 + \Gamma^2/4}$$

<u>Note</u> $\Gamma_{\Delta\to\pi^- p} = B_{\Delta\to\pi^- p}\cdot\Gamma = \frac{1}{3}\Gamma$

$\Gamma_{\Delta\to\pi^0 n} = B_{\Delta\to\pi^0 n}\,\Gamma = \frac{2}{3}\Gamma$

$E_{cm} = M_\Delta$

$$\Rightarrow \sigma = \left(\frac{2\pi}{p^2}\right)\frac{\frac{2}{9}\Gamma^2}{\Gamma^2/4} = \frac{16\pi}{9p^2} \otimes (\hbar c)^2$$

↳ to get right units (barns)

<u>p?</u>

$\xrightarrow[\pi^-]{} \; \xleftarrow[p]{}$ in <u>C.M. frame</u>:

(1) $E_\pi + E_p = M_\Delta$ (E-conservation)

(2) $|P_\pi| = |P_p| \equiv P$ (mom-conservation)

(2) $P_\pi^2 = P_p^2 \Leftrightarrow P_\pi^2 + m_\pi^2 + m_p^2 = P_p^2 + m_p^2 + m_\pi^2$

$\Leftrightarrow E_\pi^2 + m_p^2 = E_p^2 + m_\pi^2$

$\overset{+(1)}{\Leftrightarrow} \cancel{E_\pi^2} + m_p^2 = (M_\Delta - E_\pi)^2 + m_\pi^2$

$= M_\Delta^2 + \cancel{E_\pi^2} - 2M_\Delta E_\pi + m_\pi^2$

$$\Rightarrow E_\pi = \frac{M_\Delta^2 + m_\pi^2 - m_p^2}{2M_\Delta} = 266.7 \text{ MeV}$$

$$\Rightarrow P_\pi = P_p = p = \sqrt{E_\pi^2 - m_\pi^2} = 227.7 \text{ MeV}$$

($1b = 100\,fm^2$)

$$\Rightarrow \sigma = \frac{16\pi}{9(227.7)^2\,MeV^2}\cdot(197)^2\,MeV^2 fm^2 = 4.2\,fm^2 = 4.2\cdot10^{-2}\text{ barns}$$

$= \underline{42\text{ mb}}$

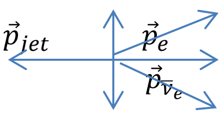
IIa

$$E_{cm} = \sqrt{s} = 270 + 270 = 540\ GeV = E_{jet} + E_W$$
$$\vec{p}_{jet} + \vec{p}_W = 0$$

$$E_{cm} = E_{jet} + E_W = p_W + \sqrt{p_W^2 + m_W^2} \to (E_{cm} - p_W)^2 = p_W^2 \ + m_W^2$$

$$p_W = \frac{E_{cm}^2 - m_W^2}{2E_{cm}}$$
$$\vec{p}_W = \vec{p}_e + \vec{p}_{\bar{\nu}_e}$$

Energy and momentum of electron and neutrino must be equal for maximum opening angle.



Use invariant mass to get the opening angle

$$m_W^2 = (E_\nu + E_e)^2 - (\vec{p}_e + \vec{p}_\nu)^2$$

$$|p_e| = |p_\nu| = E_e = E_\nu = \frac{E_W}{2} = \frac{E_{cm} - |p_W|}{2} = \frac{E_{cm} - \frac{E_{cm}^2 - m_W^2}{2E_{cm}}}{2} = \frac{E_{cm}^2 + m_W^2}{4E_{cm}} = x$$
$$m_W^2 = (2x)^2 - (2x^2 + 2x^2 \cos\theta_{e\nu}) = 2x^2(1 - \cos\theta_{e\nu})$$
$$\theta_{e\nu} = \mathrm{acos}\left(1 - \frac{m_W^2}{2x^2}\right) = \mathrm{acos}\left(1 - \frac{8E_{cm}^2 m_W^2}{(E_{cm}^2 + m_W^2)^2}\right) = \mathrm{acos}(1 - \frac{8 * 540^2 * 80^2}{(540^2 + 80^2)^2}) = 34^\circ$$

IIb (for answers on b and c see pages 230 and 231 of book)

The fact that $W \to l^+ \nu_l$ has the same branching for all $l = e, \mu, \tau \approx 10.8\%$

IIc

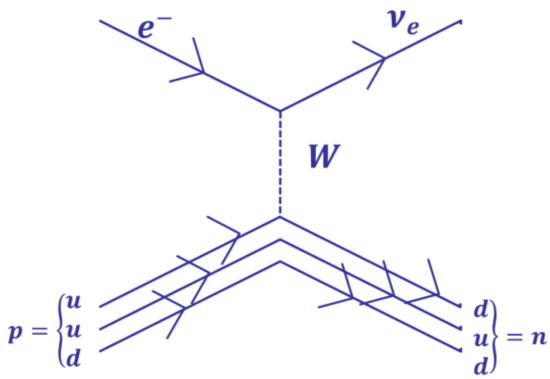
$W^+ \to u\bar{d}'$ or $c\bar{s}'$and not $t\bar{b}'(m_t > m_W)$ each quark pair has three possibilities (color) means 6 possibilities to make hadrons as compared to a lepton pair 6*10.8%=65% is close to the number in the PDG datasheet (68% , we ignore all details of phase space and higher order diagrams)

The Cabbibo angle does not play a role the generation does not enter in describing the $d'$and s' thus we working with an approximate unitary transformation going from $(d, s) \to (d', s')$.

IId (see book p 223)

$\Gamma_{l\nu_l} = \alpha_W M_W$=10.8% of 2.08 GeV $\alpha_W = \frac{0.108\times2.08}{80} = 2.79 \times 10^{-3}\ or\ \frac{1}{\alpha_w} = 360$

IIIa



Determine energy threshold:

$$E_e + m_p = E_\nu + E_n \xrightarrow{threshold} E_n \approx m_n + \frac{p_e^2}{2m_n}$$
$$\vec{p}_e = \vec{p}_\nu + \vec{p}_n \xrightarrow{threshold} \vec{p}_n$$

Ignoring recoil threshold is $E_e = m_n - m_p = 2.933$ MeV with recoil solve quadratic equation

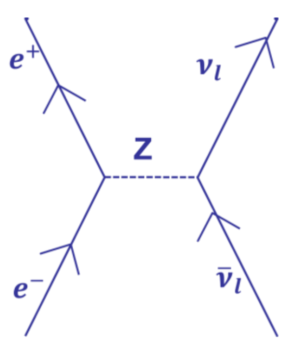
$$E_e + m_p = m_n + \frac{E_e^2 - m_e^2}{2m_n}$$

$E_e = m_n - m_p - \frac{m_e^2}{2m_n} = 2.932$ MeV

Electron neutrino is born and propagates as mass eigensstates. May assume independent propagations (incoherent) therefore $0.85^2\ m_1\ and\ 0.53^2\ m_2\ and\ 0\ m_3$ note that total probability is 1 (one). When they arrive on Earth
$m_1\ has\ probability\ 0.85^2\ to\ be\ an\ electron\ neutrino, m_2\ has\ 0.53^2\ electron\ probability$

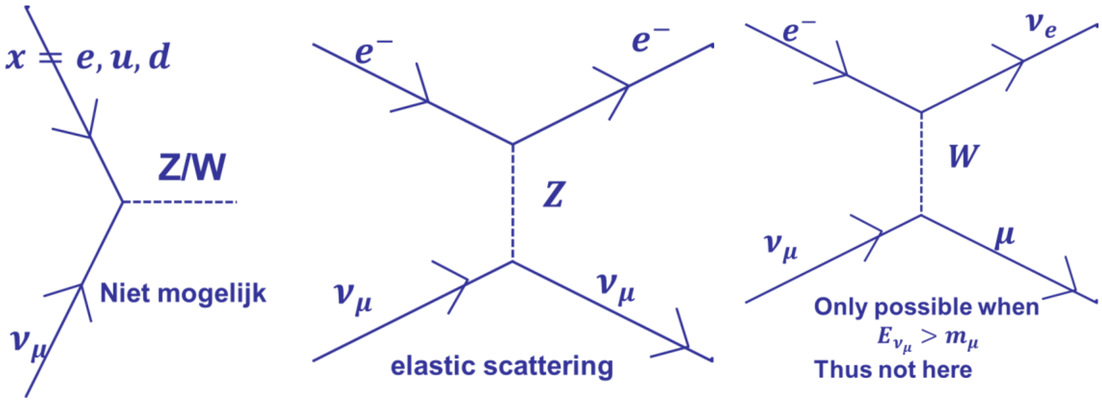
Thus we have a probability $0.85^4 + 0.53^4 = 0.60$ to be an electron neutrino.

IIIb



Equal amounts of each neutrino and antineutrino i.e. 1/6 after propagation because of unitarity also 1/6

IIIc



Only middle diagram possible.

IIId

$$\Delta t = \frac{L}{v_1} - \frac{L}{v_2} = \frac{L}{c}\left(\frac{1}{\beta_1} - \frac{1}{\beta_2}\right)$$

$$\beta = \frac{p}{E} = \frac{\sqrt{E^2 - m^2}}{E} \approx 1 - \frac{1}{2}\frac{m^2}{E^2}$$

$$\Delta t = \frac{L}{c}\left(1 + \frac{1}{2}\frac{m^2}{E_1^2} - 1 - \frac{1}{2}\frac{m^2}{E_2^2}\right) \to \frac{m^2 L}{2c}\left(\frac{1}{E_1^2} - \frac{1}{E_2^2}\right) \to m^2 = \frac{\frac{2c\Delta t}{L}}{\left(\frac{1}{E_1^2} - \frac{1}{E_2^2}\right)}$$

$$= 2 \times \frac{5.3}{5.3} \times 10^{-12} \times \left(1 - \frac{1}{9}\right)[MeV]^2 \to m = \sqrt{\frac{18}{8}} \approx 1.4\ eV$$

IIIe p330

$< m >= \sum_i |U_{ei}|^2 m_i$ (the weighted sum (sum of weights is one) of probabilities that m_i contributes)

IIId

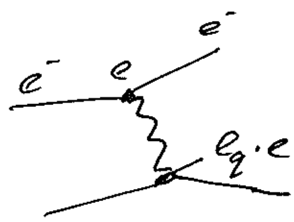
$$< m^2 >= \sum_i |U_{ei}|^2 m_i^2$$

Same argument but for m_i^2.

(4)

1. δ-function: elastic scattering of electron on particle with mass $m \Rightarrow x = \frac{Q^2}{2m\nu} = 1$ (E & p conservation)
Hence, δ function guarantees that $\underline{\underline{x = 1}}$. Note, $Q^2 = M^2 + 2M\nu - W^2$
For elastic scattering $M = W$
$\Rightarrow Q^2 = 2M\nu$

e_q^2: follows from $\alpha\ (\sim e^2) \longrightarrow e_q \alpha$, since:



2. for a parton i:

$$W_2^i = e_i^2\, \delta\left(\nu - \frac{Q^2}{2zM_p}\right)$$

$\Rightarrow$ incoherent sum with probability density $f_i(z)$:

$$W_2 = \sum_i \int e_i^2 \underbrace{\delta\left(\nu - \frac{Q^2}{2zM_p}\right)} f_i(z)\, dz$$

$$= \delta\left[\frac{\nu}{z}\left(z - \frac{Q^2}{2M_p\nu}\right)\right]$$

$$= \delta\left[\frac{\nu}{z}(z - x)\right]$$

$$= \frac{z}{\nu}\,\delta(z - x)$$

$$W_2 = \sum_i \int e_i^2 \frac{z}{\nu}\,\delta(z-x) f_i(z)\, dz = \frac{1}{\nu} \sum_i e_i^2 \int z f_i(z)\, \delta(z-x)\, dz$$

$$= \frac{1}{\nu} \sum_i e_i^2\, x f_i(x)$$

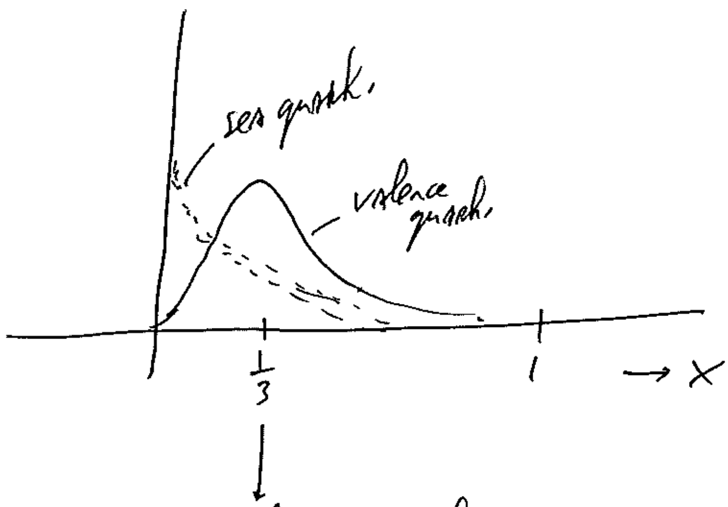
$$\Rightarrow \nu \cdot W_2 = \sum_i e_i^2\, x f_i(x) \equiv F_2(x)$$

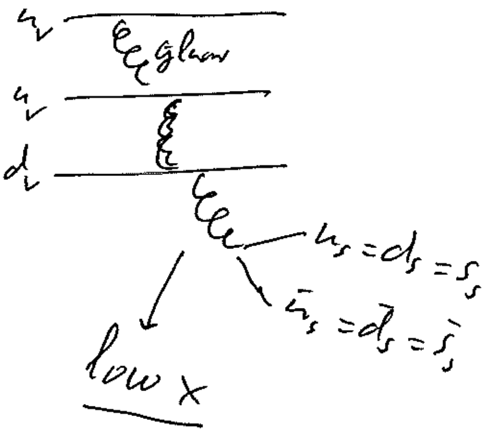
(4) cont'd

3. The ~~[illegible]~~ "formfactor" function F_2 (also $F_1 = M_p W_1$) only depends upon 1 variable, x, independent on Q^2!

$\Rightarrow$ parton is <u>point-like</u>

4.





valence quarks around $\frac{1}{3}$, but smeared due to gluon exchange.